Lectures In Functional Analysis And Operator Theory Graduate Texts In Mathematics

As long as a branch of knowledge offers an abundance of problems, it is full of vitality. David Hilbert Over the last 15 years I have given lectures on a variety of problems in nonlinear functional analysis and its applications. In doing this, I have recommended to my students a number of excellent monographs devoted to specialized topics, but there was no complete survey-type exposition of nonlinear functional analysis making available a quick survey to the wide range of readers including mathematicians, natural scientists, and engineers who have only an elementary knowledge of linear functional analysis. I have tried to close this gap with my five-part lecture notes, the first three parts of which have been published in the Teubner-Texte series by Teubner-Verlag, Leipzig, 1976, 1977, and 1978. The present English edition was translated from a completely rewritten manuscript which is significantly longer than the original version in the Teubner-Texte series. The material is organized in the following way: Part I: Fixed Point Theorems. Part II: Monotone Operators. Part III: Variational Methods and Optimization. Parts IV jV: Applications to Mathematical Physics. The exposition is guided by the following considerations: (a) What are the supporting basic ideas and what intrinsic interrelations exist between them? (/3) In what relation do the basic ideas stand to the known propositions of classical analysis and linear functional analysis? (y) What typical applications are there? VII Preface viii Special emphasis is placed on motivation.

This textbook provides a careful treatment of functional analysis and some of its applications in analysis, number theory, and ergodic theory. In addition to discussing core material in functional analysis, the authors cover more recent and advanced topics, including Weyl's law for eigenfunctions of the Laplace operator, amenability and property (T), the measurable functional calculus, spectral theory for unbounded operators, and an account of Tao's approach to the prime number theorem using Banach algebras. The book further contains numerous examples and exercises, making it suitable for both lecture courses and self-study. Functional Analysis, Spectral Theory, and Applications is aimed at postgraduate and advanced undergraduate students with some background in analysis and algebra, but will also appeal to everyone with an interest in seeing how functional analysis can be applied to other parts of mathematics.

This textbook is a completely revised, updated, and expanded English edition of the important Analyse fonctionnelle (1983). In addition, it contains a wealth of problems and exercises (with solutions) to guide the reader. Uniquely, this book presents in a coherent, concise and unified way the main results from functional analysis together with the main results from the theory of partial differential equations (PDEs). Although there are many books on functional analysis and many on PDEs, this is the first to cover both of these closely connected topics. Since the French book was first published, it has been translated into Spanish,

Italian, Japanese, Korean, Romanian, Greek and Chinese. The English edition makes a welcome addition to this list.

This book is based on lectures given at "Mekhmat", the Department of Mechanics and Mathematics at Moscow State University, one of the top mathematical departments worldwide, with a rich tradition of teaching functional analysis. Featuring an advanced course on real and functional analysis, the book presents not only core material traditionally included in university courses of different levels, but also a survey of the most important results of a more subtle nature, which cannot be considered basic but which are useful for applications. Further, it includes several hundred exercises of varying difficulty with tips and references. The book is intended for graduate and PhD students studying real and functional analysis as well as mathematicians and physicists whose research is related to functional analysis.

This book is intended for those having only a moderate background in mathematics, who need to increase their mathematical knowledge for development in their areas of work and to read the related mathematical literature. The material covered, which includes practically all the information on functional analysis that may be necessary for those working in various areas of applications of mathematics, as well as the simplicity of presentation, differentiates this book from others. About 300 examples and more than 500 problems are provided to help readers understand and master the theories presented. The list of references enables readers to explore those topics in which they are interested, and gather further information about applications used as examples in the book.Applications: Probability Theory and Statistics, Signal and Image Processing, Systems Analysis and Design.

"This book covers such topics as Lp? spaces, distributions, Baire category, probability theory and Brownian motion, several complex variables and oscillatory integrals in Fourier analysis. The authors focus on key results in each area, highlighting their importance and the organic unity of the subject"--Provided by publisher.

This textbook is addressed to graduate students in mathematics or other disciplines who wish to understand the essential concepts of functional analysis and their applications to partial differential equations. The book is intentionally concise, presenting all the fundamental concepts and results but omitting the more specialized topics. Enough of the theory of Sobolev spaces and semigroups of linear operators is included as needed to develop significant applications to elliptic, parabolic, and hyperbolic PDEs. Throughout the book, care has been taken to explain the connections between theorems in functional analysis and familiar results of finite-dimensional linear algebra. The main concepts and ideas used in the proofs are illustrated with a large number of figures. A rich collection of homework problems is included at the end of most chapters. The book is suitable as a text for a one-semester graduate course. Nonlinearity and Functional Analysis is a collection of lectures that aim to present

a systematic description of fundamental nonlinear results and their applicability to a variety of concrete problems taken from various fields of mathematical analysis. For decades, great mathematical interest has focused on problems associated with linear operators and the extension of the well-known results of linear algebra to an infinite-dimensional context. This interest has been crowned with deep insights, and the substantial theory that has been developed has had a profound influence throughout the mathematical sciences. This volume comprises six chapters and begins by presenting some background material, such as differential-geometric sources, sources in mathematical physics, and sources from the calculus of variations, before delving into the subject of nonlinear operators. The following chapters then discuss local analysis of a single mapping and parameter dependent perturbation phenomena before going into analysis in the large. The final chapters conclude the collection with a discussion of global theories for general nonlinear operators and critical point theory for gradient mappings. This book will be of interest to practitioners in the fields of mathematics and physics, and to those with interest in conventional linear functional analysis and ordinary and partial differential equations.

This volume is not only intended for mathematicians who deal with applications of functional analysis, but also for those having only a moderate background in mathematics in their areas of work.The materials covered, which includes practically all the information on functional analysis that may be necessary for those working in various areas of mathematics applications, as well as the simplicity of presentation, differentiates this book from others. The method and style of presentation of materials make it digestible and easily understood by readers.This second edition includes new and updated 300 examples and more than 500 problems to help readers understand and master the theories presented. In addition, necessary improvements for bringing the contents more up to date with current fundamental and applied developments in Chapters 1-10 were made. Now, Chapter 9 covers nonlinear and stochastic problems and Chapter 10, devoted to elements of numerical functional analysis, has been completely revised and broadened.

Functional analysis is a broad mathematical area with strong connections to many domains within mathematics and physics. This book, based on a first-year graduate course taught by Robert J. Zimmer at the University of Chicago, is a complete, concise presentation of fundamental ideas and theorems of functional analysis. It introduces essential notions and results from many areas of mathematics to which functional analysis makes important contributions, and it demonstrates the unity of perspective and technique made possible by the functional analytic approach. Zimmer provides an introductory chapter summarizing measure theory and the elementary theory of Banach and Hilbert spaces, followed by a discussion of various examples of topological vector spaces, seminorms defining them, and natural classes of linear operators. He then presents basic results for a wide range of topics: convexity and fixed point

theorems, compact operators, compact groups and their representations, spectral theory of bounded operators, ergodic theory, commutative C*-algebras, Fourier transforms, Sobolev embedding theorems, distributions, and elliptic differential operators. In treating all of these topics, Zimmer's emphasis is not on the development of all related machinery or on encyclopedic coverage but rather on the direct, complete presentation of central theorems and the structural framework and examples needed to understand them. Sets of exercises are included at the end of each chapter. For graduate students and researchers in mathematics who have mastered elementary analysis, this book is an entrée and reference to the full range of theory and applications in which functional analysis plays a part. For physics students and researchers interested in these topics, the lectures supply a thorough mathematical grounding.

Numerous detailed proofs highlight this treatment of functional equations. Starting with equations that can be solved by simple substitutions, the book then moves to equations with several unknown functions and methods of reduction to differential and integral equations. Also includes composite equations, equations with several unknown functions of several variables, vector and matrix equations, more. 1966 edition.

A NATO Advanced Study Institute on Nonlinear Functional Analysis and Its Applications was held in Hotel Villa del Mare, Maratea, It.a1y during April 22 - May 3, 1985. This volume consists of the Proceedings of the Institute. These Proceedings include the invited lectures and contributed papers given during the Institute. The papers have been refereed. The aim of these lectures was to bring together recent and up-to-date development of the subject, and to give directions for future research. The main topics covered include: degree and generalized degree theory, results related to Hamiltonian Systems, Fixed Point theory, linear and nonlinear Differential and Partial Differential Equations, Theory of Nielsen Numbers, and applications to Dynamical Systems, Bifurcation Theory, Hamiltonian Systems, Minimax Theory, Heat Equations, Pendulum Equation, Nonlinear Boundary Value Problems, and Dirichlet and Neumann problems for elliptic equations and the periodic Dirichlet problem for semilinear beam equations. I express my sincere thanks to Professors F. E. Browder, R. Conti, A. Do1d, D. E. Edmunds and J. Mawhin members of the Advisory Committee.

A. Balakrishnan: A constructive approach to optimal control.- R. Glowinski: Méthodes itératives duales pour la minimisation de fonctionnelles convexes.- J.L. Lions: Approximation numérique des inéquations d'évolution.- G. Marchuk: Introduction to the methods of numerical analysis.- U. Mosco: An introduction to the approximate solution of variational inequalities.- I. Singer: Best approximation in normed linear spaces.- G. Strang: A Fourier analysis of the finite element variational method.- M. Zerner: Caractéristiques d'approximation des compacts dans les espaces fonctionnels et problèmes aux limites elliptiques.

With this second volume, we enter the intriguing world of complex analysis. From the first theorems on, the elegance and sweep of the results is evident. The starting point is the simple idea of extending a function initially given for real values of the argument to one that is defined when the argument is complex. From there, one proceeds to the main properties of holomorphic functions, whose proofs are generally short and quite illuminating: the Cauchy theorems, residues, analytic continuation, the argument

principle. With this background, the reader is ready to learn a wealth of additional material connecting the subject with other areas of mathematics: the Fourier transform treated by contour integration, the zeta function and the prime number theorem, and an introduction to elliptic functions culminating in their application to combinatorics and number theory. Thoroughly developing a subject with many ramifications, while striking a careful balance between conceptual insights and the technical underpinnings of rigorous analysis, Complex Analysis will be welcomed by students of mathematics, physics, engineering and other sciences. The Princeton Lectures in Analysis represents a sustained effort to introduce the core areas of mathematical analysis while also illustrating the organic unity between them. Numerous examples and applications throughout its four planned volumes, of which Complex Analysis is the second, highlight the far-reaching consequences of certain ideas in analysis to other fields of mathematics and a variety of sciences. Stein and Shakarchi move from an introduction addressing Fourier series and integrals to in-depth considerations of complex analysis; measure and integration theory, and Hilbert spaces; and, finally, further topics such as functional analysis, distributions and elements of probability theory.

In the early 1960s, by using techniques from the model theory of first-order logic, Robinson gave a rigorous formulation and extension of Leibniz'' infinitesimal calculus. Since then, the methodology has found applications in a wide spectrum of areas in mathematics, with particular success in the probability theory and functional analysis. In the latter, fruitful results were produced with Luxemburg''s invention of the nonstandard hull construction. However, there is still no publication of a coherent and self-contained treatment of functional analysis using methods from nonstandard analysis. This publication aims to fill this gap.

This is the fourth and final volume in the Princeton Lectures in Analysis, a series of textbooks that aim to present, in an integrated manner, the core areas of analysis. Beginning with the basic facts of functional analysis, this volume looks at Banach spaces, Lp spaces, and distribution theory, and highlights their roles in harmonic analysis. The authors then use the Baire category theorem to illustrate several points, including the existence of Besicovitch sets. The second half of the book introduces readers to other central topics in analysis, such as probability theory and Brownian motion, which culminates in the solution of Dirichlet's problem. The concluding chapters explore several complex variables and oscillatory integrals in Fourier analysis, and illustrate applications to such diverse areas as nonlinear dispersion equations and the problem of counting lattice points. Throughout the book, the authors focus on key results in each area and stress the organic unity of the subject. A comprehensive and authoritative text that treats some of the main topics of modern analysis A look at basic functional analysis and its applications in harmonic analysis, probability theory, and several complex variables Key results in each area discussed in relation to other areas of mathematics Highlights the organic unity of large areas of analysis traditionally split into subfields Interesting exercises and problems illustrate ideas Clear proofs provided

This book provides readers with a concise introduction to current studies on operator-algebras and their generalizations, operator spaces and operator systems, with a special focus on their application in quantum information science. This basic framework for the mathematical formulation of quantum information can be traced back to the mathematical work of John von Neumann, one of the pioneers of operator algebras, which forms the underpinning of most current mathematical treatments of the quantum theory, besides being one of the most dynamic areas of twentieth century functional analysis. Today, von Neumann's foresight finds expression in the rapidly growing field of quantum information theory. These notes gather the

content of lectures given by a very distinguished group of mathematicians and quantum information theorists, held at the IMSc in Chennai some years ago, and great care has been taken to present the material as a primer on the subject matter. Starting from the basic definitions of operator spaces and operator systems, this text proceeds to discuss several important theorems including Stinespring's dilation theorem for completely positive maps and Kirchberg's theorem on tensor products of C*-algebras. It also takes a closer look at the abstract characterization of operator systems and, motivated by the requirements of different tensor products in quantum information theory, the theory of tensor products in operator systems is discussed in detail. On the quantum information side, the book offers a rigorous treatment of quantifying entanglement in bipartite quantum systems, and moves on to review four different areas in which ideas from the theory of operator systems and operator algebras play a natural role: the issue of zero-error communication over quantum channels, the strong subadditivity property of quantum entropy, the different norms on quantum states and the corresponding induced norms on quantum channels, and, lastly, the applications of matrix-valued random variables in the quantum information setting.

These Proceedings form a record of the lectures presented at the interna tional Conference on Functional Analysis and Approximation held at the Ober wolfach Mathematical Research Institute, August 9-16, 1980. They include 33 of the 38 invited conference papers, as well as three papers subsequently submitted in writing. Further, there is a report devoted to new and unsolved problems, based on two special sessions of the conference. The present volume is the sixth Oberwolfach Conference in Birkhauser's ISNM series to be edited at Aachen *. It is once again devoted to more significant results obtained in the wide areas of approximation theory, harmonic analysis, functional analysis, and operator theory during the past three years. Many of the papers solicited not only outline fundamental advances in their fields but also focus on interconnections between the various research areas. The papers in the present volume have been grouped into nine chapters. Chapter I, on operator theory, deals with maps on positive semidefinite opera tors, spectral bounds of semigroup operators, evolution equations of diffusion type, the spectral theory of propagators, and generalized inverses. Chapter II, on functional analysis, contains papers on modular approximation, interpolation spaces, and unconditional bases.

The Book Is Intended To Serve As A Textbook For An Introductory Course In Functional Analysis For The Senior Undergraduate And Graduate Students. It Can Also Be Useful For The Senior Students Of Applied Mathematics, Statistics, Operations Research, Engineering And Theoretical Physics. The Text Starts With A Chapter On Preliminaries Discussing Basic Concepts And Results Which Would Be Taken For Granted Later In The Book. This Is Followed By Chapters On Normed And Banach Spaces, Bounded Linear Operators, Bounded Linear Functionals. The Concept And Specific Geometry Of Hilbert Spaces, Functionals And Operators On Hilbert Spaces And Introduction To Spectral Theory. An Appendix Has Been Given On Schauder Bases.The Salient Features Of The Book Are: * Presentation Of The Subject In A Natural Way * Description Of The Concepts With Justification * Clear And Precise Exposition Avoiding Pendantry * Various Examples And Counter Examples * Graded Problems Throughout Each ChapterNotes And Remarks Within The Text Enhances The Utility Of The Book For The Students.

The book is based on courses taught by the author at Moscow State University. Compared to many other books on the subject, it is unique in that the exposition is based on extensive use of the language and elementary constructions of category theory. Among topics featured in the book are the theory of Banach and Hilbert tensor products, the theory of distributions and weak topologies, and Borel operator calculus. The book contains many examples illustrating the general theory presented, as well as multiple exercises that help the reader to learn the subject. It can be used as a textbook on selected topics of functional analysis and operator

theory. Prerequisites include linear algebra, elements of real analysis, and elements of the theory of metric spaces.

This textbook, based on three series of lectures held by the author at the University of Strasbourg, presents functional analysis in a non-traditional way by generalizing elementary theorems of plane geometry to spaces of arbitrary dimension. This approach leads naturally to the basic notions and theorems. Most results are illustrated by the small lp spaces. The Lebesgue integral, meanwhile, is treated via the direct approach of Frigyes Riesz, whose constructive definition of measurable functions leads to optimal, clear-cut versions of the classical theorems of Fubini-Tonelli and Radon-Nikodým. Lectures on Functional Analysis and the Lebesgue Integral presents the most important topics for students, with short, elegant proofs. The exposition style follows the Hungarian mathematical tradition of Paul Erd?s and others. The order of the first two parts, functional analysis and the Lebesgue integral, may be reversed. In the third and final part they are combined to study various spaces of continuous and integrable functions. Several beautiful, but almost forgotten, classical theorems are also included. Both undergraduate and graduate students in pure and applied mathematics, physics and engineering will find this textbook useful. Only basic topological notions and results are used and various simple but pertinent examples and exercises illustrate the usefulness and optimality of most theorems. Many of these examples are new or difficult to localize in the literature, and the original sources of most notions and results are indicated to help the reader understand the genesis and development of the field.

This book is intended for those having only a moderate background in mathematics, who need to increase their mathematical knowledge for development in their areas of work and to read the related mathematical literature. The material covered, which includes practically all the information on functional analysis that may be necessary for those working in various areas of applications of mathematics, as well as the simplicity of presentation, differentiates this book from others. About 300 examples and more than 500 problems are provided to help readers understand and master the theories presented. The list of references enables readers to explore those topics in which they are interested, and gather further information about applications used as examples in the book. Applications: Probability Theory and Statistics, Signal and Image Processing, Systems Analysis and Design.

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